Lunar Laser Ranging

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1 Motivation and Introduction

On July 20th 1969 the USA reported about the first successful Moon landing. A short time later they published pictures of the laser retro-reflectors and reported about successful range measurements. The first measurements apart from August 1st 1969 obviously showed that the retro-reflector had sustained the low lunar night temperatures and worked perfectly [A11_PSR_1969 [1] page 166f].
Since then one hears and reads again and again from such laser range measurements onto lunar laser retro-reflectors.
By the way the very first lunar laser range measurement had taken place within the project *Luna See* of the MIT already in 1962 [The laser in astronomy, New Scientist [12]]; there the laser beam had been reflected or scattered by the lunar surface only.

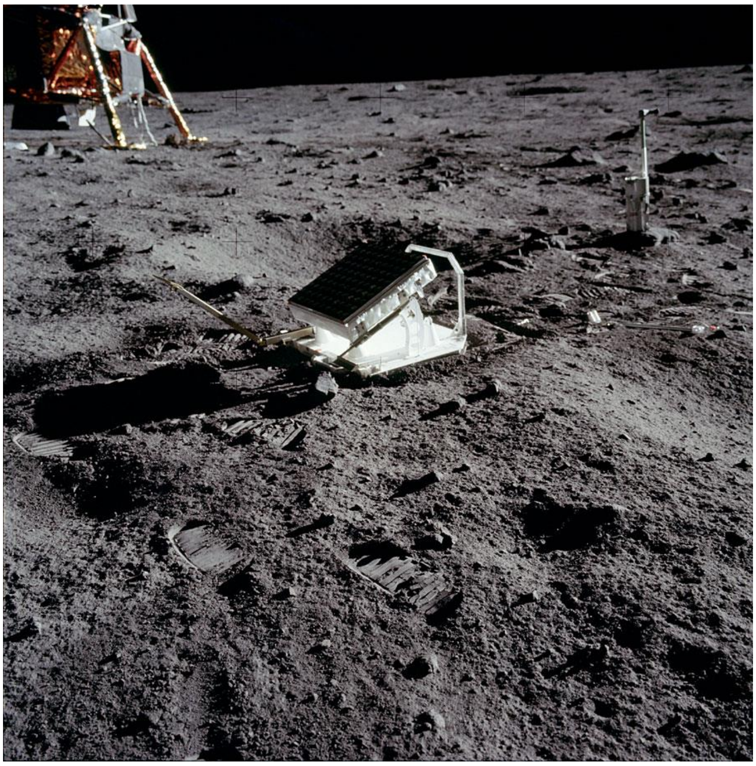

Figure 1-1 Laser Retro Reflector of Apollo 11 (AS11-40-5952.jpg, [3])

Here these results shall be checked about their authenticity.

2 Measurement Principle and Applications

In the lunar laser ranging the time of flight of a laser pulse is measured. The laser pulse flies from the sender or transmitter on the Earth to the Moon and back:

$$z = \frac{c \cdot \Delta T}{2} \qquad (1)$$

with z: Distance between measurement station and measuring object (Moon)
c: Velocity of light ≈ 300‘000 km/s
ΔT: Time of flight of the laser pulse

The following applications operate according to the same principle:

- Laser altimeters in aircrafts, which measure the altitude above ground
- Laser altimeters which measure the altitude of a space craft above the surface of a celestial body and so determine its ground profile
- Satellite Laser Ranging (SLR): Measurement of the distance from a ground station to a satellite, e.g. LAGEOS (Laser Geodynamics Satellite)

Figure 2-1 LAGEOS, made specifically for SLR [Wikipedia: LAGEOS]

Another measurement principle with lasers is the interferometry. It is applied mostly for short distances where a high accuracy is required. The interferometry is not further discussed here because it is not used for the lunar laser ranging.

To increase the return signals one often uses retro-reflectors. They reflect the light exactly into the direction of the incoming light.
Laser altimeters of aircrafts and of spacecrafts measure without retro reflectors. In the SLR all satellites to be measured are equipped with retro reflectors.
The fact that there is also a back scattered signal even without any retro-reflector can easily be demonstrated with a laser pointer which is directed to a wall: the spot is well visible.
Measurements to the Moon can principally be made with or without retro-reflectors. But for accurate long term measurements a retroreflector is compulsory because only then one can be sure that the measurements are done to the same reference point.

3 Theory

The emitted laser pulse must have sufficient energy or a sufficient number of photons so that at least one photon returns to the receiver.
“One photon” sounds little, but today’s receivers can detect single photons with a high probability (>70%) (e.g. [4]). Since there are efficient receivers for the green light one generally selects the wave length of 532nm. 532nm is half of 1064nm which is the wave length of NdYAG lasers.

Here the difference between measurements with and without retro reflectors shall be considered. For both the link budget shall be derived.

3.1 Link-Budget for Measurements without any Retro-Reflector

In this measurement principle the laser light is scattered on the surface of the measurement object (here: surface of the Moon). The scattering is modelled as a Lambertian scattering where the light is scattered in all directions. Only a small part of the light of a surface element is therefore scattered back in the direction of the incoming light; but light is scattered back from the whole illuminated area. The field of view of the receiver is big enough so that it can collect light from the whole illuminated area.

The link budget is calculated according to [BELA_2007, [5], formula (5)] for vacuum:

$$E_{RX} = E_{TX} \cdot T_{RX} \cdot \frac{A_{RX}}{z^2} \cdot \frac{Albedo}{\pi} \qquad \text{(2a)}$$

with
- E_{RX}: Received energy of one pulse
- E_{TX}: Emitted energy of one pulse
- T_{RX}: Transmission of the receiver telescope
- A_{RX}: Area of the receiver telescope
- z: Distance from the sender to the measurement object
- Albedo: Reflectivity of the measured surface

For a measurement through the atmosphere one has additionally to consider the atmospheric transmission T_{atm} for the run forward and backward. If the receiver telescope has a circular entrance, A_{RX} can be written as $R_{RX}^2 \cdot \pi$, if a potential central obscuration is neglected. If the emitted power is measured before the transmit telescope then its transmission (T_{TX}) has to be considered as well. Additionally the quantum efficiency η_q (≈0.7) of the detector is introduced. The adapted equation (2a) looks then as follows:

$$\frac{E_{RX}}{E_{TX}} = \eta_q \cdot T_{TX} \cdot T_{RX} \cdot T_{atm}^{\ 2} \cdot \frac{R_{RX}^{\ 2}}{z^2} \cdot Albedo \qquad \text{(2b)}$$

For numerical values for all transmissions 0.707 is taken. These values seem on the one hand reasonable and simplify on the other hand the calculation, because $0.707^2=0.5$. For the atmospheric transmission this is valid according to [Degnan_1993 [6] Fig.7] for quite good weather conditions. For the optical transmission through a telescope also 0.707 is assumed; this value considers mainly the central obscuration and for the transmit telescope additionally the Gaussian beam profile [Klein_Degnan_1974 [7] Fig.3]. The albedo of the Moon is ≈0.1, and the distance from the Earth to the Moon is set to 380’000km (measurement distance, not distance of the centres).
So (2c) shows the link budget of the lunar ranging with numerical values:

$$\frac{E_{RX}}{E_{TX}} = 0.7 \cdot 0.5 \cdot 0.5 \cdot \frac{R_{RX}^{\ 2}}{(380'000km)^2} \cdot 0.1 = 0.0175 \cdot \frac{R_{RX}^{\ 2}}{(380'000km)^2} \qquad \text{(2c)}$$

3.2 Link-Budget for Measurements with Retro-Reflectors

At this measurement principle the light which hits the retro reflectors is mainly reflected back along the direction of the incoming light. But the longer the measurement distance, the less light hits the retro reflectors. Therefore this principle is not a priori better than a measurement with back scattered light.

The link budget is set up according to [Degnan_1993 [6] equation 3.1.1] for a homogenous atmosphere and perpendicular incidence of the light on the retroreflector:

$$n_{RX} = \eta_q \cdot n_{TX} \cdot T_{TX} \cdot G_{TX} \cdot \sigma_{RR} \cdot \left(\frac{1}{4 \cdot \pi \cdot z^2} \right)^2 \cdot A_{RX} \cdot T_{RX} \cdot T_{atm}^{\ 2} \cdot T_{Cir}^{\ 2} \qquad (3)$$

with
- n_{RX}: Number of received photons
- η_q: Quantum efficiency of the detector (receiver)
- n_{TX}: Number of emitted photons
- T_{TX}: Transmission of the transmit telescope
- G_{TX}: Antenna gain of the transmitter
- σ_{RR}: Optical cross section of a single retro reflector (cube corner) (see below)
- z: Distance from the sender to the measurement object
- A_{RX}: Area of the receiver telescope
- T_{RX}: Transmission of the receiver telescope
- T_{atm}: One way transmission through the atmosphere
- T_{Cir}: One way transmission through possible cirrus clouds

T_{Cir} is set to 1, i.e. it is assumed that there are no disturbing cirrus clouds during the measurement.

σ_{RR} is defined in [Degnan_1993 [6] equation 6.1.1] for a single retro reflector as follows:

$$\sigma_{RR} = R_{RR} \cdot \frac{4 \cdot \pi \cdot A_{RR}}{\Omega_{RR}} = R_{RR} \cdot \frac{4 \cdot \pi \cdot A_{RR}^{\ 2}}{\lambda^2} \qquad (4)$$

with
- R_{RR}: Reflectivity of a retro reflector (≥ 0.9; in the following I assume 0.9)
- A_{RR}: Area of a retro reflector ($=D_{RR}^2 \cdot \pi/4$)
- Ω_{RR}: Illuminated solid angle of a retro reflector
- λ: Wave length: here 532 nm (green)

Equation (3) is expanded as follows:

1. σ_{RR} is inserted according to equation (4)
2. for G_{TX} the diffraction limited gain of $\frac{4 \cdot \pi \cdot A_{TX}}{\lambda^2}$ is inserted
3. an additional efficiency factor η_{add} is introduced
4. it is considered, that there are several (n_{RR}) retro reflectors present:

$$\frac{n_{RX}}{n_{TX}} = \eta_{add} \cdot \eta_q \cdot T_{TX} \cdot n_{RR} \cdot R_{RR} \cdot \frac{4 \cdot \pi \cdot A_{TX}}{\lambda^2} \cdot \frac{4 \cdot \pi \cdot A_{RR}^{\ 2}}{\lambda^2} \cdot \left(\frac{1}{4 \cdot \pi \cdot z^2} \right)^2 \cdot A_{RX} \cdot T_{RX} \cdot T_{atm}^{\ 2} \qquad (5a)$$

$$\frac{n_{RX}}{n_{TX}} = \eta_{add} \cdot \eta_q \cdot T_{TX} \cdot T_{RX} \cdot T_{atm}^{\ 2} \cdot n_{RR} \cdot R_{RR} \cdot \frac{A_{TX} \cdot A_{RR}^{\ 2} \cdot A_{RX}}{\lambda^4 \cdot z^4} \qquad (5b)$$

Equation (5b) is still too idealized because the transmit beam, which uses only a part of A_{RX} (i.e. $A_{TX} < A_{RX}$) or which is intentionally expanded, could be further expanded by the (turbulent) atmosphere and so the transmit gain would be reduced. Instead of the real antenna area an antenna with a radius equal to the transverse atmospheric coherence length ρ_0 is selected – on the basis of [Degnan_1993 [6] equation 3.9.9]. Instead of ρ_0 one can often see the Fried parameter r_0 which is about $2\cdot\rho_0$. ρ_0 is for good weather conditions >10cm [WHT Seeing, [10]]; but for a ground station at sea level the value is smaller, i.e. about 1..2cm:

$$\frac{n_{RX}}{n_{TX}} = \eta_{add} \cdot \eta_q \cdot T_{TX} \cdot T_{RX} \cdot T_{atm}{}^2 \cdot n_{RR} \cdot R_{RR} \cdot \frac{(\rho_0{}^2 \cdot \pi) \cdot A_{RR}{}^2 \cdot A_{RX}}{\lambda^4 \cdot z^4} \quad (6)$$

The retro reflector of Apollo 11 is an array of 100 (n_{RR}) single reflectors, each with a diameter (D_{RR}) of 3.8cm [A11_PSR_1969 [1] page 167].
For η_q and the transmissions the same numerical values as in §3.1 are taken.

Equation (6) is now the basis to calculate the number of received photons. The following parameters shall be varied:

ρ_0: Transverse atmospheric coherence length (10cm and 2cm). If $R_{TX} < \rho_0$, then R_{TX} would replace ρ_0. But 10cm corresponds to a divergence of 0.7'' which is still larger than the divergence used in Apache Point, which is < 0.5''.

η_{add}: Additional efficiency factor, to consider possible neglects (1 and 0.5)

The velocity aberration is neglected, because it is significantly smaller than the beam angle of the retro reflector [A11_PSR_1969 [1] page 167]. This and other influences can be investigated with the additional efficiency factor η_{add} (1 ... ≈0.5).

4 Expected and Measured Values

The following three Lunar Laser Ranging (LLR) stations

- Apache Point Observatory, 2'788m altitude, USA
- Wettzell, 600m altitude, Germany
- Observatory of the Côte d'Azur, 1'270m altitude, France

as well as the estimation in [Dickey 1994 [2] page 5f]
and the LIDAR measurments *Luna See* in 1962 are compared.

First the number of received photons per pulse is calculated according to the equations (2c) and (6). Table 4-1 shows the expected number of photons for a measurement without retro reflectors, Table 4-2 shows the number of reflected photons – reflected on the retro reflector. During a measurement on a retro reflector the scattered photons are measured as well.

Table 4-1 Expected Number of Scattered Photons (Scattered on the Lunar Surface)

	Apache Point	**Wettzell**	**∅1m Telescope**	**Côte d'Azur**	***Luna See***
Telescope-∅	3.5 m	0.75 m	1 m	1.5 m	1.22 m
Transmitted number of photons per pulse	$3{\cdot}10^{17}$	10^{19} (1)	10^{21} (2)	$8{\cdot}10^{17}$	$1.75{\cdot}10^{20}$
Expected number of received photons (Equation 2c)	**0.11**	**0.17**	**30**	**0.06**	**8**

(1) Pulse chain, consisting of several single pulses
(2) Virtual example, see also Table 4-2

Table 4-2 Expected Number of Reflected Photons (Reflected on the Retro-Reflectors)

	Parameter		Apache Point	Wettzell	∅1m Telescope	Côte d'Azur
	η_{add}	ρ_0				
Telescope-∅			3.5 m	0.75 m	1 m	1.5 m
Transmitted number of photons per pulse			$3 \cdot 10^{17}$	10^{19} (1)	10^{21} (2)	$8 \cdot 10^{17}$
Expected number of received photons (Equation 6)	1	0.1 m	**1'100**	**1'680**	**300'000**	**540**
	0.5	0.1 m	**550**	**840**	**150'000**	**270**
	0.5	0.02 m	**22**	**34**	**6'000**	**11 (3)**

(1) Pulse chain, consisting of several single pulses
(2) Virtual example: [Dickey 1994 [2] page 5f] predicts here a loss of 10^{-21}, he expects therefore one single photon instead of the here minimum calculated 6'000
(3) The estimation in [11] is 32 (attenuation of $4 \cdot 10^{-17}$ - without atmosphere). This fits well with the value of 11, which considers the atmospheric attenuation with a factor of 2.

In Table 4-2 two values are parameterized, i.e. at good weather conditions the measurement values are expected to be between or above these extreme values.
The following table shows the measurement results of Apache Point, Wettzell (here only approximate results) and of the Côte d'Azur:

Table 4-3 Measured Number of Scattered and Reflected Photons

	Apache Point	Wettzell	Côte d'Azur	*Luna See*
Telescope-∅	3.5 m	0.75 m	1.5 m	1.22 m
Transmitted number of photons per pulse	$3 \cdot 10^{17}$	10^{19} (1)	$8 \cdot 10^{17}$	$1.75 \cdot 10^{20}$
Measured number of received photons per pulse	**0.1087 (2)**	**< 1**	**≈ 0.01 (6)**	**12 (7)**
	0.135 (3)			
	0.25 (4)			
	0.6 (5)			

(1) Pulse chain, consisting of several single pulses
(2) [APOLLO_2007 [8] Fig.10]
(3) [APOLLO_2007 [8] §8] „record returns" in October 2005
(4) [APOLLO_2007 [8] §8] „rates in subsequent months"
(5) [APOLLO_2007 [8] §8] „peaks on Apollo 15 array"
(6) Less than expected from the scattering. This can be caused by a (too) short range gate (or measurement depth or shutter speed).
(7) Scattering only: ≈ as calculated above

The measured values correspond well with the expected number of scattered photons. Therefore no amplification of the return signal by the retro reflector array could be measured.
The first three Apache Point measurements in Table 4-3 are within a factor of about 2 in the expected range of 0.11. The 0.135 photons/pulse have been marked as "record returns", i.e. previous measurements have obviously been less fruitful; it is not said whether this marking was absolute or relative to the telescope size.
The short-time peaks of 0.6 photons per pulse are a factor of 5.5 above the scattering budget, but still a factor 110[1] below of what would have been expected for the homing on the retro reflector array of Apollo 15.

[1] 110 = 3 · 22 / 0.6; The Apollo 15 retro reflector array consists of 300 cube corners, it is therefore 3 times larger than the one of Apollo 11, for which the budget was made.

A measured peak of a factor of 5.5 above the scattering budget would be high, but it can still be within the uncertainty. On the one hand specifically the atmospheric transmission can also be better; on the other hand the Moon's albedo is not constant over the whole surface of the Moon. Possibly such a measurement has been made on a surface with a higher reflectivity and additionally one could have benefited from the opposition surge, i.e. from an increase of the albedo if the illumination direction coincides with the direction of observation. This effect is based on the fact that the whole observed area is illuminated, that there are no visible shadows – contrary to the general constellation at which the albedo is determined.

At the measurement [APOLLO_2007 [8] Fig.10] the variation of the measured distance is very small, i.e. similar as expected for a measurement on a retro reflector. Such an effect can be achieved if one measures on a surface perpendicular to the measurement direction. This is indeed possible, because the beam angle is according to [Degnan_1993 [6] equation 3.9.9] $\pm\frac{\lambda}{\pi \cdot \rho_0}$. This corresponds with ρ_0=2cm to a radius on the Moon of 3.2 km; with ρ_0=10cm the radius is only 640m and the corresponding spot area is 1.3 km^2.

As already the introduction of the additional efficiency factor η_{add} and the not simple estimation of the influence of the atmosphere show, a precise prediction of the number of received photons for a measurement on retro reflectors on the Moon is not trivial.
A possibly important point has not been addressed yet: a flat retro reflector array is sensitive on the incident beam direction. The back reflected light drops very fast, if the incident beam direction is not perpendicular. This is shown in [Degnan_1993 [6] Fig.23]: at a deviation of the incident beam direction of 13° only 50% is reflected back and at a deviation of ≥40° nothing at all is reflected back.
But if one knew the position of the retro reflectors so exactly including their construction, one could expect that this would be considered in the link budgets or that the measurements would be made when the incident angle would be more or less perpendicular.

5 Conclusion

Even if these laser distance measurements are reported to have been made onto Lunar Retro-Reflectors, an accordingly amplified return signal has never been measured. With other words: one had only measured the scattered light from the lunar surface. The analyses here and of the observatory of the Côte d'Azur show clearly that the return signal from a retro-reflector would be much higher.
The reflector based Lunar Laser Ranging is obviously one part of the spectacle to present the Apollo Moon landings as real. The stories around this thematic are well invented:

- The retro reflectors were still operating normally after 25 years [Dickey 1994 [2] p. 3]
- The distance from Earth to the Moon increases by 3.8 cm per year [Dickey 1994 [2] page 34]

The value of 3.8 cm per year could be completely wrong because one does not know where exactly on the Moon the light spot had been reflected. To compensate for this uncertainty one would have to measure over a much longer time.

Critical challenges of top-class published results have obviously no place in the today's scientific world.
In [APOLLO_2007 [8] §8] the inconsistency of the expected link budget is addressed, but immediately played down: *„... we have not seen returns at the rate anticipated by a simple link budget...*". As shown above, the link budget is not simple at all, specifically if one wants to fill in sound numerical values.

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